

DIASTATIC ENTROPY AND RIGIDITY OF HYPERBOLIC MANIFOLDS

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ABSTRACT. Let $f : Y \rightarrow X$ be a continuous map between a compact real analytic Kähler manifold (Y, g) and a compact complex hyperbolic manifold (X, g_0) . In this paper we give a lower bound of the diastatic entropy of (Y, g) in terms of the diastatic entropy of (X, g_0) and the degree of f . When the lower bound is attained we get geometric rigidity theorems for the diastatic entropy analogous to the ones obtained by G. Besson, G. Courtois and S. Gallot [2] for the volume entropy. As a corollary, when $X = Y$, we show that the minimal diastatic entropy is achieved if and only if g is isometric to the hyperbolic metric g_0 .

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1. INTRODUCTION AND STATEMENT OF MAIN RESULTS

In this paper, we define the *diastatic entropy* $\text{Ent}_d(Y, g)$ of a compact real analytic Kähler manifold (Y, g) with *globally defined diastasis function* (see Definition 2.1 and 2.2 below). This is a real analytic invariant defined, in the noncompact case, by the author in [17], where the link with Donaldson's balanced condition is studied. The diastatic entropy extends the concept of volume entropy using the diastasis function instead of the geodesic distance. Throughout this paper a compact *complex hyperbolic manifold* will be a compact real analytic complex manifold (X, g_0) endowed with locally Hermitian symmetric metric with holomorphic sectional curvature strictly negative (i.e. (X, g_0) is the compact quotient of a complex hyperbolic space, see Example 2.3 below). Our main result is the following theorem, analogous to the celebrated result of G. Besson, G. Courtois, S. Gallot on the

minimal *volume entropy* of a compact negatively curved locally symmetric manifold (see (12) below) [2, Théorème Principal]:

Theorem 1.1. *Let (Y, g) be a compact Kähler manifold of dimension $n \geq 2$ and let (X, g_0) be a compact complex hyperbolic manifold of the same dimension. If $f : Y \rightarrow X$ is a nonzero degree continuous map, then*

$$\text{Ent}_d(Y, g)^{2n} \text{Vol}(Y, g) \geq |\deg(f)| \text{Ent}_d(X, g_0)^{2n} \text{Vol}(X, g_0). \quad (1)$$

Moreover the equality is attained if and only if f is homotopic to a holomorphic or anti-holomorphic homothetic¹ covering $F : Y \rightarrow X$.

As a first corollary we obtain a characterization of the hyperbolic metric as that metric which realizes the minimum of the diastatic entropy:

Corollary 1.1. *Let (X, g_0) be a compact complex hyperbolic manifold of dimension $n \geq 2$ and denote by $\mathcal{E}(X, g_0)$ the set of metrics g on X with globally defined diastasis and fixed volume $\text{Vol}(g) = \text{Vol}(g_0)$. Then the functional $\mathcal{F} : \mathcal{E}(X, g_0) \rightarrow \mathbb{R} \cup \{\infty\}$ given by $g \mapsto \text{Ent}_d(X, g)$, attains its minimum when g is holomorphically or anti-holomorphically isometric to g_0 .*

This corollary can be seen as the *diastatic* version of the A. Katok and M. Gromov conjecture on the minimal *volume entropy* of a locally symmetric space with strictly negative curvature (see [8, p. 58]), proved by G. Besson, G. Courtois, S. Gallot in [2]. We also apply Theorem 1.1 to give a simple proof for the complex version of the Mostow and Corlette–Siu–Thurston rigidity theorems:

Corollary 1.2. *(Mostow). Let (X, g_0) and (Y, g) be two compact complex hyperbolic manifolds of dimension $n \geq 2$. If X and Y are homotopically equivalent then they are holomorphically or anti-holomorphically homothetic.*

Corollary 1.3. *(Corlette–Siu–Thurston). Let (X, g_0) and (Y, g) be as in the previous corollary and with the same (constant) holomorphic sectional curvature. If $f : Y \rightarrow X$ is a continuous map such that*

$$\text{Vol}(Y) = |\deg(f)| \text{Vol}(X) \quad (2)$$

then there exists a holomorphically or anti-holomorphically Riemannian covering $F : Y \rightarrow X$ homotopic to f .

The paper consists of others two sections. In Section 2 we recall the basic definitions. Section 3 is dedicated to the proof of Theorem 1.1. The proof is based on the analogous result for the volume entropy (see formula (12) below) and on

¹ F is said to be homothetic if $F^*g_0 = \alpha g$ for some $\alpha > 0$.

Lemma 3.2 which provides a lower bound for the diastatic entropy in terms of volume entropy.

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2. DIASTASIS AND DIASTASIC ENTROPY

The diastasis is a special Kähler potential defined by E. Calabi in its seminal paper [5]. Let (\tilde{Y}, \tilde{g}) be a real analytic Kähler manifold. For every point $p \in \tilde{Y}$ there exists a real analytic function $\Phi : V \rightarrow \mathbb{R}$, called Kähler potential, defined in a neighborhood V of p such that $\tilde{\omega} = \frac{i}{2} \partial \bar{\partial} \Phi$, where $\tilde{\omega}$ is the Kähler form associated to \tilde{g} . Let $z = (z_1, \dots, z_n)$ be a local coordinates system around p . By duplicating the variables z and \bar{z} the real analytic Kähler potential Φ can be complex analytically continued to a function $\hat{\Phi} : U \times U \rightarrow \mathbb{C}$ in a neighbourhood $U \times U \subset V \times V$ of (p, p) which is holomorphic in the first entry and antiholomorphic in the second one.

Definition 2.1 (Calabi, [5]). The *diastasis function* $\mathcal{D} : U \times U \rightarrow \mathbb{R}$ is defined by

$$\mathcal{D}(z, w) := \hat{\Phi}(z, \bar{z}) + \hat{\Phi}(w, \bar{w}) - \hat{\Phi}(z, \bar{w}) - \hat{\Phi}(w, \bar{z}).$$

The *diastasis function centered in w* , is the Kähler potential $\mathcal{D}_w : U \rightarrow \mathbb{R}$ around w given by

$$\mathcal{D}_w(z) := \mathcal{D}(z, w).$$

We will say that a compact Kähler manifold (Y, g) has *globally defined diastasis* if its universal Kähler covering (\tilde{Y}, \tilde{g}) has globally defined diastasis $\mathcal{D} : \tilde{Y} \times \tilde{Y} \rightarrow \mathbb{R}$.

One can prove that the diastasis is uniquely determined by the Kähler metric \tilde{g} and that it does not depend on the choice of the local coordinates system or on the choice of the Kähler potential Φ .

Calabi in [5] uses the diastasis to give necessary and sufficient conditions for the existence of an holomorphic isometric immersion of a real analytic Kähler manifolds into a complex space form. For others interesting applications of the diastasis function see [10, 11, 12, 13, 14, 15, 18] and reference therein.

Assume that (\tilde{Y}, \tilde{g}) has globally defined diastasis $\mathcal{D} : \tilde{Y} \times \tilde{Y} \rightarrow \mathbb{R}$. Its (normalized²) diastatic entropy is defined by:

$$\text{Ent}_d(\tilde{Y}, \tilde{g}) = \mathcal{X}(\tilde{g}) \inf \left\{ c \in \mathbb{R}^+ : \int_{\tilde{Y}} e^{-c \mathcal{D}_w} \nu_{\tilde{g}} < \infty \right\}, \quad (3)$$

where $\mathcal{X}(\tilde{g}) = \sup_{y, z \in \tilde{Y}} \|\text{grad}_y \mathcal{D}_z\|$ and $\nu_{\tilde{g}}$ is the volume form associated to \tilde{g} . If $\mathcal{X}(\tilde{g}) = \infty$ or the infimum in (3) is not achieved by any $c \in \mathbb{R}^+$, we set

²Our definition of diastatic entropy differs respect to the one given in [17] by the normalizing factor $\mathcal{X}(\tilde{g})$.

$\text{Ent}_d(\tilde{Y}, \tilde{g}) = \infty$. The definition does not depend on the base point w , indeed, as

$$|\mathcal{D}_{w_1}(x) - \mathcal{D}_{w_2}(x)| = |\mathcal{D}_x(w_1) - \mathcal{D}_x(w_2)| \leq \mathcal{X}(\tilde{g}) \rho(w_1, w_2),$$

we have

$$e^{-c \mathcal{X}(\tilde{g}) \rho(w_1, w_2)} \int_{\tilde{Y}} e^{-c \mathcal{D}_{w_1}(x)} \nu_{\tilde{g}} \leq \int_{\tilde{Y}} e^{-c \mathcal{D}_{w_2}(x)} \nu_{\tilde{g}} \leq e^{c \mathcal{X}(\tilde{g}) \rho(w_1, w_2)} \int_{\tilde{Y}} e^{-c \mathcal{D}_{w_1}(x)} \nu_{\tilde{g}},$$

therefore $\int_{\tilde{Y}} e^{-c \mathcal{D}_{w_2}(x)} \nu_{\tilde{g}} < \infty$ if and only if $\int e^{-c \mathcal{D}_{w_1}(x)} \nu_{\tilde{g}} < \infty$.

Definition 2.2. Let (Y, g) be a compact Kähler manifold with globally defined diastasis. We define the *diastatic entropy* of (Y, g) as

$$\text{Ent}_d(Y, g) = \text{Ent}_d(\tilde{Y}, \tilde{g}),$$

where (\tilde{Y}, \tilde{g}) is the universal Kähler covering of (Y, g) .

Example 2.3. Let $\mathbb{C}H^n = \{z \in \mathbb{C}^n : \|z\|^2 = |z_1|^2 + \dots + |z_n|^2 < 1\}$ be the unitary disc endowed with the hyperbolic metric \tilde{g}_h of constant holomorphic sectional curvature -4 . The associated Kähler form and the diastasis are respectively given by

$$\tilde{\omega}_h = -\frac{i}{2} \partial \bar{\partial} \log(1 - \|z\|^2).$$

and

$$\mathcal{D}^h(w, z) = -\log \left(\frac{(1 - \|z\|^2)(1 - \|w\|^2)}{|1 - zw^*|^2} \right). \quad (4)$$

Denote by $\omega_e = \frac{i}{2} \partial \bar{\partial} \|z\|^2$ the restriction to $\mathbb{C}H^n$ of the flat form of \mathbb{C}^n . One has

$$\int_{\mathbb{C}H^n} e^{-\alpha \mathcal{D}_0^h} \frac{\omega_h^n}{n!} = \int_{\mathbb{C}H^n} (1 - |z|^2)^{\alpha - n - 1} \frac{\omega_e^n}{n!} < \infty \Leftrightarrow \alpha > n,$$

and by a straightforward computation one sees that $\mathcal{X}(\tilde{g}_h) = 2$. We conclude by (3) that

$$\text{Ent}_d(\mathbb{C}H^n, \tilde{g}_h) = 2n. \quad (5)$$

Remark 2.4. It should be interesting to compute $\mathcal{X}(g_B)$, where g_B is the Bergman metric of an homogeneous bounded domain. This combined with the results obtained in [17], will allow us to obtain the diastatic entropy of this domains.

3. PROOF OF THEOREM 1.1 AND COROLLARIES 1.1, 1.2 AND 1.3

We start by recalling the definition of *volume entropy* of a compact Riemannian manifold (M, g) . Let $\pi : (\tilde{M}, \tilde{g}) \rightarrow (M, g)$ its riemannian universal cover. We define the volume entropy of (M, g) as

$$\text{Ent}_v(M, g) = \inf \left\{ c \in \mathbb{R}^+ : \int_{\tilde{M}} e^{-c \tilde{\rho}(w, x)} \nu_{\tilde{g}}(x) < \infty \right\}, \quad (6)$$

where $\tilde{\rho}$ is the geodesic distance on $(\widetilde{M}, \tilde{g})$ and $\nu_{\tilde{g}}$ is the volume form associated to \tilde{g} . By the triangular inequality, we can see that the definition does not depend on the base point w . As the volume entropy depends only on the Riemannian universal cover it make sense to define

$$\text{Ent}_v(\widetilde{M}, \tilde{g}) = \text{Ent}_v(M, g).$$

The *classical definition* of volume entropy of a compact riemannian manifold (M, g) , is the following

$$\text{Ent}_{\text{vol}}(M, g) = \lim_{t \rightarrow \infty} \frac{1}{t} \log \text{Vol}(B_p(t)), \quad (7)$$

where $\text{Vol}(B_p(t))$ denotes the volume of the geodesic ball $B_p(t) \subset \widetilde{M}$, of center in p and radius t . This notion of entropy is related with one of the main invariant for the dynamics of the geodesic flow of (M, g) : the topological entropy $\text{Ent}_{\text{top}}(M, g)$ of this flow. For every compact manifold (M, g) A. Manning in [19] proved the inequality $\text{Ent}_{\text{vol}}(M, g) \leq \text{Ent}_{\text{top}}(M, g)$, which is an equality when the curvature is negative. We refer the reader to the paper [2] (see also [3] and [4]) of G. Besson, G. Courtois and S. Gallot for an overview on the volume entropy and for the proof of the celebrated minimal entropy theorem. For an explicit computation of the volume entropy $\text{Ent}_v(\Omega, g)$ of a symmetric bounded domain (Ω, g) see [16].

The next lemma shows that the classical definition of volume entropy (7) does not depend on the base point and it is equivalent to definition (6), that is

$$\text{Ent}_{\text{vol}}(M, g) = \text{Ent}_v(M, g).$$

Lemma 3.1. *Denote by*

$$\underline{L} := \liminf_{R \rightarrow +\infty} \left(\frac{1}{R} \log(\text{Vol } B(x_0, R)) \right)$$

and

$$\overline{L} := \limsup_{R \rightarrow +\infty} \left(\frac{1}{R} \log(\text{Vol } B(x_0, R)) \right),$$

where $B(x_0, R) \subset (\widetilde{M}, \tilde{g})$ is the geodesic ball of centre x_0 and radius R . Then the two limits does not depends on x_0 and

$$\underline{L} \leq \text{Ent}_v(M, g) \leq \overline{L}.$$

Proof. Let x_1 an arbitrary point of M . Set $D = d(x_0, x_1)$ and $R > D$. By the triangular inequality

$$B(x_0, R - D) \subset B(x_1, R) \subset B(x_0, R + D).$$

Let $R' = R + D$, we have

$$\begin{aligned} \liminf_{R \rightarrow +\infty} \left(\frac{1}{R} \log (\text{Vol } B(x_1, R)) \right) &\leq \liminf_{R \rightarrow +\infty} \left(\frac{1}{R} \log (\text{Vol } B(x_0, R + D)) \right) \\ &= \liminf_{R' \rightarrow +\infty} \left(\frac{R'}{R' - D} \frac{1}{R'} \log (\text{Vol } B(x_0, R')) \right) \\ &\leq \liminf_{R' \rightarrow +\infty} \left(\frac{1}{R'} \log (\text{Vol } B(x_0, R')) \right). \end{aligned}$$

With the same argument one can prove the inequality in the other direction, so that \underline{L} does not depend on x_0 . Analogously we can prove that \overline{L} does not depend on x_0 .

By the definition of limit inferior and superior, for every $\varepsilon > 0$, there exists $R_0(\varepsilon)$ such that, for $R \geq R_0(\varepsilon)$,

$$\underline{L} - \varepsilon \leq \left(\frac{1}{R} \log (\text{Vol } B(x_0, R)) \right) \leq \overline{L} + \varepsilon$$

equivalently

$$e^{(\underline{L} - \varepsilon)R} \leq (\text{Vol } B(x_0, R)) \leq e^{(\overline{L} + \varepsilon)R}. \quad (8)$$

Integrating by parts we obtain

$$\begin{aligned} I &:= \int_{\widehat{M}} e^{-c} \tilde{\rho}(x_0, x) dv(x) = \int_0^\infty e^{-cr} \text{Vol}_{n-1}(S(x_0, r)) dr \\ &= \text{Vol}(B(x_0, r)) e^{-cr} \Big|_0^\infty + c \int_0^\infty e^{-cr} \text{Vol}(B(x_0, r)) dr. \end{aligned}$$

where $S(x_0, r) = \partial B(x_0, r)$. On the other hand, by (8) we get

$$\int_{R_0(\varepsilon)}^\infty e^{(\underline{L} - c - \varepsilon)r} dr \leq \int_{R_0(\varepsilon)}^\infty e^{-cr} \text{Vol}(B(x_0, r)) dr \leq \int_{R_0(\varepsilon)}^\infty e^{-(c - \overline{L} - \varepsilon)r} dr.$$

We deduce that if $c > \overline{L}$ then I is convergent i.e $\overline{L} \geq \text{Ent}_v$ and that if I is not convergent when $c < \underline{L}$, that is $\text{Ent}_v \geq \underline{L}$, as wished. \square

The next lemma show that the diastatic entropy is bounded from below by the volume entropy.

Lemma 3.2. *Let (Y, g) be a compact Kähler manifold with globally defined diastasis, then*

$$\text{Ent}_d(Y, g) \geq \text{Ent}_v(Y, g). \quad (9)$$

This bound is sharp when (Y, g) is a compact quotient of the complex hyperbolic space. That is,

$$\text{Ent}_d(\mathbb{C}H^n, \tilde{g}_h) = 2n = \text{Ent}_v(\mathbb{C}H^n, \tilde{g}_h). \quad (10)$$

Proof. Let (\tilde{Y}, \tilde{g}) be universal Kähler cover of (Y, g) . For every $w, x \in \tilde{Y}$ we have

$$\mathcal{D}_w(x) = \mathcal{D}_w(x) - \mathcal{D}_w(w) \leq \sup_{z \in \tilde{Y}} \|d_z \mathcal{D}_w\| \rho_w(x) \leq \mathcal{X}(\tilde{g}) \rho_w(x),$$

so

$$\int_{\tilde{Y}} e^{-c \mathcal{X}(\tilde{g}) \rho_w(x)} \nu_{\tilde{g}} \leq \int_{\tilde{Y}} e^{-c \mathcal{D}_w(x)} \nu_{\tilde{g}}.$$

Therefore, if $c \mathcal{X}(\tilde{g}) \leq \text{Ent}_v(\tilde{Y}, \tilde{g})$ then $c \mathcal{X}(\tilde{g}) \leq \text{Ent}_d(\tilde{Y}, \tilde{g})$. We obtain (9) by setting $c = \frac{\text{Ent}_v(\tilde{Y}, \tilde{g})}{\mathcal{X}(\tilde{g})}$. Equation (10) follow by (5) and [16, Theorem 1.1]. \square

Proof of Theorem 1.1. Let (X, g_0) as in Theorem 1.1 and let $\pi_X : (\mathbb{C}H^n, \tilde{g}_0) \rightarrow (X, g_0)$ be the universal covering. Notice that $\tilde{g}_0 = \lambda \tilde{g}_h$ for some positive λ . Then we have

$$\begin{aligned} \text{Vol}(X, g_0) \text{Ent}_v(X, g_0)^{2n} &= \text{Vol}(X, g_h) \text{Ent}_v(X, g_h)^{2n} \\ &= \text{Vol}(X, g_h) \text{Ent}_d(X, g_h)^{2n} = \text{Vol}(X, g_0) \text{Ent}_d(X, g_0)^{2n}, \end{aligned} \quad (11)$$

where the first and the third equality are consequence of the fact that $\text{Ent}_v(\mathbb{C}H^n, \tilde{g}_0) = \frac{1}{\sqrt{\lambda}} \text{Ent}_v(\mathbb{C}H^n, \tilde{g}_h)$ and $\text{Ent}_d(\mathbb{C}H^n, \tilde{g}_0) = \frac{1}{\sqrt{\lambda}} \text{Ent}_d(\mathbb{C}H^n, \tilde{g}_h)$, while the second equality follows by (10). Let $f : Y \rightarrow X$ be as in Theorem 1.1, then, by [2, Théorème Principal] we know that

$$\text{Ent}_v(Y, g)^{2n} \text{Vol}(Y, g) \geq |\deg(f)| \text{Ent}_v(X, g_0)^{2n} \text{Vol}(X, g_0) \quad (12)$$

where the equality is attained if and only if f is homotopic to a homothetic covering $F : Y \rightarrow X$. Putting together (9), (11) and (12) we get that

$$\text{Ent}_d(Y, g)^{2n} \text{Vol}(Y, g) \geq |\deg(f)| \text{Ent}_d(X, g_0)^{2n} \text{Vol}(X, g_0)$$

where the equality is attained if and only if f is homotopic to a homothetic covering $F : Y \rightarrow X$.

To conclude the proof it remains to prove that F is holomorphic or anti-holomorphic. Up to homotheties, it is not restrictive to assume that $g = F^* g_0$, so that its lift $\tilde{F} : \tilde{Y} \rightarrow \mathbb{C}H^n$ to the universal covering it is a global isometry. Fix a point $q \in \tilde{Y}$, let $p = \tilde{F}(q)$ and denote $A_q = \tilde{F}^* J_{0_p}$ the endomorphism acting on $T_q \tilde{Y}$, where J_0 is the complex structure of $\mathbb{C}H^n$. Denote by $\mathcal{G}_{\tilde{Y}}$ and respectively $\mathcal{G}_{\mathbb{C}H^n}$ the holonomy groups of (\tilde{Y}, \tilde{g}) and respectively $(\mathbb{C}H^n, \tilde{g}_0)$. Note that $\mathcal{G}_{\tilde{Y}} = \tilde{F}^* \mathcal{G}_{\mathbb{C}H^n}$ and that $\mathcal{G}_{\mathbb{C}H^n} = SU(n)$, therefore $\mathcal{G}_{\tilde{Y}}$ acts irreducibly on $T_q \tilde{Y}$. As J_0 commutes with the action of $\mathcal{G}_{\mathbb{C}H^n}$, by construction A_q is invariant with respect to the action of $\mathcal{G}_{\tilde{Y}}$. Therefore, denoted Id_q the identity map of $T_q \tilde{Y}$, by Schur's lemma, $A_q = \lambda \text{Id}_q$ with $\lambda \in \mathbb{C}$. Moreover $-\text{Id}_q = A_q^2 = \lambda^2 \text{Id}_q$, so $\lambda = \pm i$. By the arbitrariness of q we conclude that \tilde{F} is holomorphic or anti-holomorphic.

Proof of Corollary 1.1. This is an immediate consequence of Theorem 1.1 once assumed $Y = X$, $\text{Vol}(g) = \text{Vol}(g_0)$ and $f = \text{id}_X$ the identity map of X .

Proof of Corollary 1.2. Let $h : Y \rightarrow X$ be a homotopic equivalence and h^{-1} its homotopic inverse. Substituting in (1), once with $f = h$ and once with $f = h^{-1}$,

we have respectively

$$\mathrm{Ent}_d(Y, g)^{2n} \mathrm{Vol}(Y, g) \geq |\deg(h)| \mathrm{Ent}_d(X, g_0)^{2n} \mathrm{Vol}(X, g_0)$$

and

$$\mathrm{Ent}_d(X, g_0)^{2n} \mathrm{Vol}(X, g_0) \geq |\deg(h^{-1})| \mathrm{Ent}_d(Y, g)^{2n} \mathrm{Vol}(Y, g).$$

We then conclude that $\mathrm{Ent}_d(Y, g)^{2n} \mathrm{Vol}(Y, g) = \mathrm{Ent}_d(X, g_0)^{2n} \mathrm{Vol}(X, g_0)$ and that $|\deg(h)| = 1$. Therefore, by applying the last part of Theorem 1.1, we see that h is homotopic to an holomorphic (or antiholomorphic) homothety $F : X \rightarrow Y$.

Proof of Corollary 1.3. Let $\pi_Y : (\mathbb{C}H^n, \tilde{g}) \rightarrow (Y, g)$ and $\pi_X : (\mathbb{C}H^n, \tilde{g}_0) \rightarrow (X, g_0)$ be the universal coverings, since g_0 and g are both hyperbolic with the same curvature, we conclude that $\tilde{g}_0 = \tilde{g}$ and that $\mathrm{Ent}_d(X, g_0) = \mathrm{Ent}_d(Y, g)$. Therefore we get an equality in (1). Using again the last part of Theorem 1.1 we get $\mathrm{Vol}(Y) = |\deg(F)| \mathrm{Vol}(X)$ and we conclude that F is locally isometric.

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